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Two-point functions for $\mathcal{N}=4$ Konishi-like operators

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Abstract: We compute the two-point function of Konishi-like operators up to one-loop order, in $\mathcal{N}=4$ supersymmetric Yang-Mills theory. We work perturbatively in $\mathcal{N}=1$ superspace. We find the expression expected on the basis of superconformal invariance and determine the normalization of the correlator and the anomalous dimension of the operators to order g^2 in the coupling constant.

Keywords: Renormalization Regularization and Renormalons, Conformal and W Symmetry, Extended Supersymmetry, Superspaces.

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Recently the $\mathcal{N}=4$ supersymmetric Yang-Mills theory has attracted a lot of attention primarily in connection with the AdS/CFT correspondence [1]. Actually this theory is interesting by itself, being the prototype of a superconformal field theory in 4 dimensions [2], and there is the hope that the problem of solving such a theory might be an attackable one.

In the past $\mathcal{N}=1$ superspace techniques have been used in order to capture relevant insights on the structure of this theory [3]–[8], e.g. the computation of correlation functions or the evaluation of the anomalous dimensions of conformal operators as functions of the coupling constant in perturbation theory. Of course the aim would be to obtain exact results, i.e. one would like to determine the full dependence on the parameters that appear in the theory. Despite the fact that one deals with a perturbative expansion there is growing evidence that in certain sectors of the theory one might be able to rearrange the perturbative calculation and succeed in summing the series (see, e.g., [9] and references therein).

Our project is to see how far we can get combining these standard superspace Feynman rules with general knowledge about the conformal invariance of the theory. In this paper, as a first simple application, we want to test these techniques in the calculation of one-loop two-point functions for a class of non-protected operators which are a generalization of the well studied Konishi operator. The general form of the two-point function is fixed by conformal invariance. We will show how to obtain the logarithmic part and also the part corresponding to the normalization of the two-point function, which in general is also renormalized, up to order g^2 in the coupling.

The $\mathcal{N}=4$ supersymmetric Yang-Mills classical action written in terms of $\mathcal{N}=1$ superfields (we use the notations and conventions adopted in [10]) is given by

$$S = \mathrm{Tr}\Bigg(\int d^4x\; d^4\theta\, e^{-gV}\bar{\Phi}_i\, e^{gV}\Phi^i + \frac{1}{2g^2}\int d^4x\; d^2\theta\; W^\alpha W_\alpha + \frac{1}{2g^2}\int d^4x\; d^2\bar{\theta}\; \bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} + \\ + \frac{g}{3!}\int d^4x\; d^2\theta\; i\epsilon_{ijk}\,\Phi^i\left[\Phi^j,\Phi^k\right] + \frac{g}{3!}\int d^4x\; d^2\bar{\theta}\; i\epsilon^{ijk}\,\bar{\Phi}_i\left[\bar{\Phi}_j,\bar{\Phi}_k\right]\Bigg) \tag{1}$$

where the Φ^i with $i=1,2,3$ are three chiral superfields, and the $W^\alpha = i\bar{D}^2\left(e^{-gV}D^\alpha\, e^{gV}\right)$ are the gauge superfield strengths. All the fields are Lie-algebra valued, e.g. $\Phi^i = \Phi^i_a T^a$, in the adjoint representation of the gauge group.

We consider the following operators, for $k \geq 1$

$$\mathcal{O} = \mathrm{Tr}\left(\sum_{i=1}^{3} e^{-gV}\bar{\Phi}_i e^{gV}\Phi^{(i}\Phi^{j_1}\cdots\Phi^{j_{k-1})}\right) \qquad \bar{\mathcal{O}} = \mathrm{Tr}\left(\sum_{i=1}^{3} e^{gV}\Phi^i e^{-gV}\bar{\Phi}_{(i}\bar{\Phi}_{j'_1}\cdots\bar{\Phi}_{j'_{k-1})}\right). \tag{2}$$

We want to compute the correlators

$$\mathcal{K}\left(z,z'\right) \equiv \left\langle \mathcal{O}\left(x,\theta\right)\bar{\mathcal{O}}\left(x',\theta'\right)\right\rangle \tag{3}$$

to order g^2 in the coupling constant.

$\mathcal{N}=4$ super Yang-Mills is a superconformal field theory and we know that superconformal invariance fixes the form of the two- and three-point functions of primary operators, up to a normalization constant, in terms of their dimensions and chiral weights [11]. For the two-point function, a general expression was given in terms of $\mathcal{N}=1$ superfields in [8]

$$\left\langle \mathcal{Q}(z)\,\bar{\mathcal{Q}}(z')\right\rangle = f_{\mathcal{Q}}\left\{\frac{1}{2}D^\alpha\bar{D}^2D_\alpha+\frac{1}{4}\frac{w}{\Delta}\left[D^\alpha,\bar{D}^{\dot\alpha}\right]i\partial_{\alpha\dot\alpha}+\frac{1}{4}\frac{\Delta^2+w^2-2\Delta}{\Delta(\Delta-1)}\Box\right\}\frac{\delta^4(\theta-\theta')}{(x-x')^{2\Delta}} \tag{4}$$

where Δ is the total dimension of the operator $\mathcal{Q}$, w is the chiral weight and $f_{\mathcal{Q}}$ is the arbitrary normalization constant. The total dimension is the sum of the classical plus the anomalous dimension, $\Delta=\Delta_0+\gamma$, while the chiral weight is not renormalized because of $\mathcal{N}=4$ supersymmetry.

Our operators in (2) have $\Delta_0=k+1$ and $w=k-1$. At one-loop order we write

$$\Delta=k+1+\gamma \qquad w=k-1 \qquad f_{\mathcal{O}}=A+Bg^2 \tag{5}$$

with $\gamma=O\left(g^2\right)$. By using these values in (4) and expanding up to order g^2 a straightforward calculation gives

$$\begin{aligned}\left\langle \mathcal{O}(z)\,\bar{\mathcal{O}}(z')\right\rangle = {} & A\left\{\bar{D}^2D^2+\frac{1}{k+1}\bar{D}^{\dot\alpha}D^\alpha i\partial_{\alpha\dot\alpha}\right\}\frac{\delta^{(4)}(\theta-\theta')}{(x-x')^{2(k+1)}}- \\ & -A\gamma\left\{\bar{D}^2D^2+\frac{1}{k+1}\bar{D}^{\dot\alpha}D^\alpha i\partial_{\alpha\dot\alpha}\right\}\frac{\delta^{(4)}(\theta-\theta')}{(x-x')^{2(k+1)}}\log\left(x-x'\right)^2+ \\ & +Bg^2\left\{\bar{D}^2D^2+\frac{1}{k+1}\bar{D}^{\dot\alpha}D^\alpha i\partial_{\alpha\dot\alpha}\right\}\frac{\delta^{(4)}(\theta-\theta')}{(x-x')^{2(k+1)}}+ \\ & +A\gamma\frac{1}{2(k+1)}\left\{\frac{k-1}{k+1}\bar{D}^{\dot\alpha}D^\alpha i\partial_{\alpha\dot\alpha}+\frac{1}{k}\Box\right\}\frac{\delta^{(4)}(\theta-\theta')}{(x-x')^{2(k+1)}}\,.\end{aligned} \tag{6}$$

The first row of this formula gives the classical part of the two-point function, while the rest corresponds to the one-loop contribution.

In this paper we will reproduce the expected expression (6) by explicitly computing the two-point function up to one-loop. To this order we will determine the normalization of the two-point function and the anomalous dimension of the operators.[1].

The calculation is most easily performed using $\mathcal{N}=1$ superspace techniques: we introduce sources in the action (1) as

$$\int d^4x\; d^4\theta\,\left(\mathcal{O}J+\bar{\mathcal{O}}\bar{J}\right) \tag{7}$$

[1]Here and in the rest of the paper, with "anomalous dimension" we simply mean the coefficient of the log term in the two-point function, which in general is not an eigenvalue of the dilatation operator due to the mixing among our operators and many others with the same classical dimension. Solving this mixing problem would be an unnecessary complication for our purposes of reproducing formula (6), and moreover this was already done in [12, 13]

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and define the generating functional in euclidean space

$$W\left[J,\bar{J}\right] = \int \mathcal{D}\Phi\ \mathcal{D}\bar{\Phi}\ \mathcal{D}V\ e^{S\left[J,\bar{J}\right]} \tag{8}$$

so that the two-point function is given by

$$\left\langle \mathcal{O}\left(z\right)\bar{\mathcal{O}}\left(z'\right)\right\rangle = \left.\frac{\delta^2 W}{\delta J\left(z\right)\delta\bar{J}\left(z'\right)}\right|_{J=\bar{J}=0} \tag{9}$$

We use perturbation theory to evaluate the contributions to $W\left[J,\bar{J}\right]$ which are quadratic in the sources, i.e.

$$W\left[J,\bar{J}\right] \rightarrow \int d^4x\, d^4x'\, d^4\theta\, J\left(x,\theta,\bar{\theta}\right)\frac{F\left(g^2\right)}{\left(x-x'\right)^{2(k+1)}}\bar{J}\left(x',\theta,\bar{\theta}\right) \tag{10}$$

As mentioned above the $x-$dependence of the result is fixed by the conformal invariance of the theory, and $F\left(g^2\right)$ is the function to be determined.

In order to obtain the result in (10) one has to consider all the two-point diagrams from $W\left[J,\bar{J}\right]$ with J and $\bar{J}$ on the external legs. The rules are standard: since at one-loop we will have to deal with divergent integrals, we find it convenient to use dimensional regularization in $d=4-2\epsilon$ $x-$space within the G-scheme [14]. We need not worry about supersymmetric dimensional reduction since, as we will see, the potentially dangerous diagrams are finite. Thus we use as superfield propagators

$$\begin{aligned}\left\langle \Phi^{ia}\left(x,\theta\right)\bar{\Phi}^b{}_j\left(x',\theta'\right)\right\rangle &= \delta^i{}_j\,\delta^{ab}\,\frac{\Gamma\left(1-\epsilon\right)}{4\pi^{2-\epsilon}}\,\frac{\delta^{(4)}\left(\theta-\theta'\right)}{\left(x-x'\right)^{2(1-\epsilon)}}\\ \left\langle V^a\left(x,\theta\right)V^b\left(x',\theta'\right)\right\rangle &= -\,\delta^{ab}\,\frac{\Gamma\left(1-\epsilon\right)}{4\pi^{2-\epsilon}}\,\frac{\delta^{(4)}\left(\theta-\theta'\right)}{\left(x-x'\right)^{2(1-\epsilon)}}\end{aligned} \tag{11}$$

while the relevant vertices are read directly from the action (1) and the expressions of the operators (2).

At tree level we obtain

$$\begin{aligned}\mathcal{K}_0\left(0,z\right) &= \frac{1}{\left(2\pi\right)^{2(k+1)}}\left(k+2\right)\left(k-1\right)!\ \delta^{j_1}_{(j'_1}\cdots\delta^{j_{k-1}}_{j'_{k-1})}\times\\ &\quad\times\mathrm{Tr}\left(T_{(a_1}\cdots T_{a_{k+1})}\right)\mathrm{Tr}\left(T_{(a_1}\cdots T_{a_{k+1})}\right)\frac{\delta^{(4)}\left(\theta\right)}{x^{2(k+1)}}\,.\end{aligned} \tag{12}$$

At one-loop the relevant supergraphs are shown in figure 1. We have not drawn diagrams which give a vanishing contribution as a result of $D-$algebra or color symmetry factors. In addition we do not consider diagrams like the one shown in figure 2 which give rise to contact terms. Notice that we have not included self-energy insertions since for the $\mathcal{N}=4$ Yang-Mills theory there are no one-loop propagator corrections.

First we perform the superspace $D-$algebra and reduce the result to a multi-loop integral, then we evaluate all factors coming from combinatorics and color structures of a given diagram.

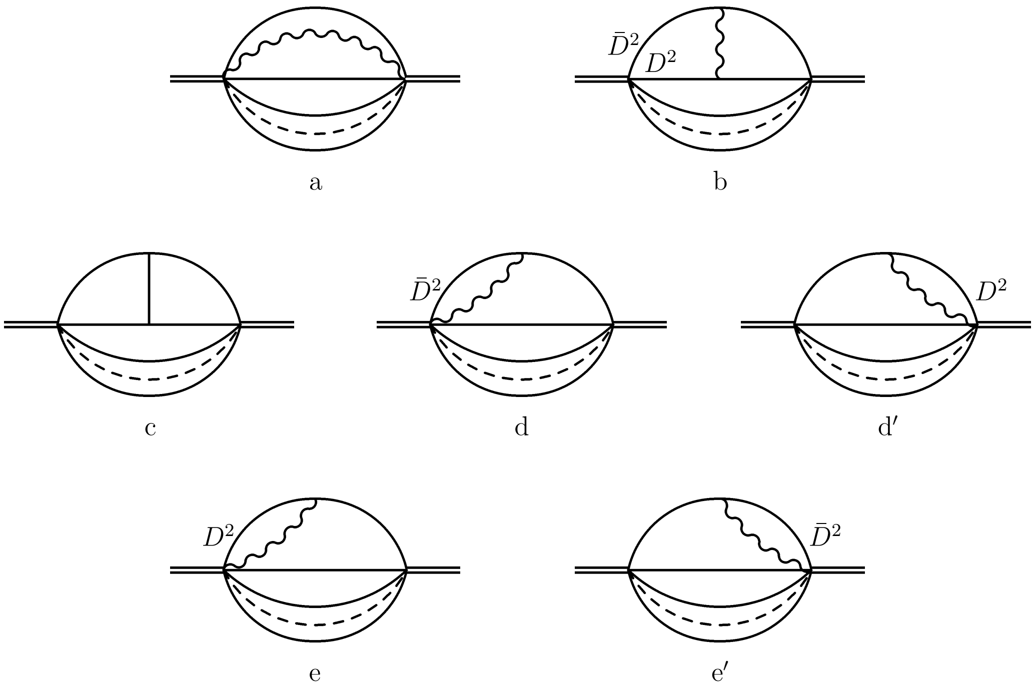

Figure 1: Diagrams whose $D-$algebra gives relevant contributions.

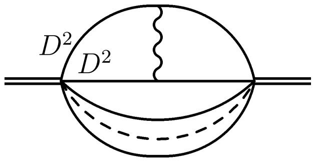

Figure 2: Diagram not vanishing, but not relevant (proportional to contact terms).

The $D-$algebra leads to the following results:

$$
\begin{aligned}
(a) &\rightarrow J\left(0,\theta,\bar{\theta}\right)\left[-I_1\left(x\right)\right]\bar{J}\left(x,\theta,\bar{\theta}\right)\\
(b) &\rightarrow J\left(0,\theta,\bar{\theta}\right)\Big[2I_2\left(x\right)\bar{D}^2D^2+2I_3^{\alpha\dot{\alpha}}\left(x\right)\bar{D}_{\dot{\alpha}}D_{\alpha}+I_4\left(x\right)\bar{D}^2D^2-\\
&\qquad -I_5^{\alpha\dot{\alpha},\beta\dot{\beta}}\left(x\right)i\partial_{\beta\dot{\beta}}\bar{D}_{\dot{\alpha}}D_{\alpha}+\left(k-1\right)I_6^{\alpha\dot{\alpha}}\left(x\right)\bar{D}_{\dot{\alpha}}D_{\alpha}\Big]\bar{J}\left(x,\theta,\bar{\theta}\right)\\
(c) &\rightarrow J\left(0,\theta,\bar{\theta}\right)\Big[I_1\left(x\right)-2I_2\left(x\right)\bar{D}^2D^2-2I_2\left(x\right)i\partial_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}D^{\alpha}+\\
&\qquad +2\,k\,I_3^{\alpha\dot{\alpha}}\left(x\right)\bar{D}_{\dot{\alpha}}D_{\alpha}+I_4\left(x\right)\bar{D}^2D^2-I_5^{\alpha\dot{\alpha},\beta\dot{\beta}}\left(x\right)i\partial_{\beta\dot{\beta}}\bar{D}_{\dot{\alpha}}D_{\alpha}+\\
&\qquad +\left(k-1\right)I_6^{\alpha\dot{\alpha}}\left(x\right)\bar{D}_{\dot{\alpha}}D_{\alpha}\Big]\bar{J}\left(x,\theta,\bar{\theta}\right)\\
(d)+(d') &\rightarrow J\left(0,\theta,\bar{\theta}\right)\left[-2I_2\left(x\right)\bar{D}^2D^2-2I_3^{\alpha\dot{\alpha}}\left(x\right)\bar{D}_{\dot{\alpha}}D_{\alpha}\right]\bar{J}\left(x,\theta,\bar{\theta}\right)\\
(e)+(e') &\rightarrow J\left(0,\theta,\bar{\theta}\right)\Big[2I_1\left(x\right)-2I_2\left(x\right)\bar{D}^2D^2-2I_2\left(x\right)i\partial_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}D^{\alpha}+\\
&\qquad +2\,k\,I_3^{\alpha\dot{\alpha}}\left(x\right)\bar{D}_{\dot{\alpha}}D_{\alpha}\Big]\bar{J}\left(x,\theta,\bar{\theta}\right)
\end{aligned}
\tag{13}
$$

where $I_1,\ldots,I_6^{\alpha\dot{\alpha}}$ correspond to the bosonic graphs in figure 3.

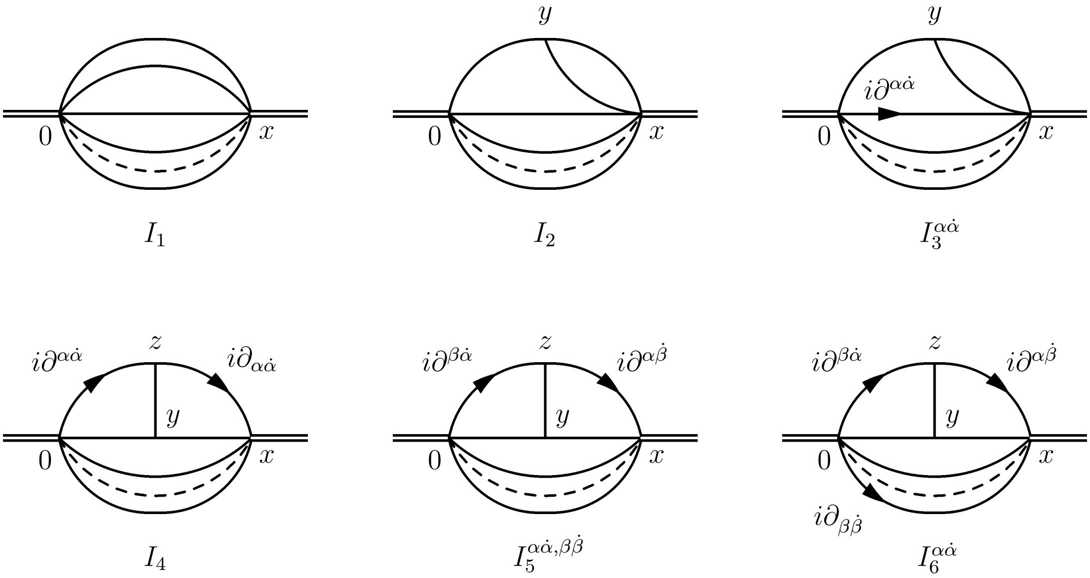

Figure 3: Bosonic contributions obtained after $D-$algebra.

They can be written explicitly using dimensional regularization in $d = 4 - 2\epsilon$ $x-$space:

$$
\begin{aligned}
I_1 &= \left(\frac{\Gamma(1-\epsilon)}{4\pi^{2-\epsilon}}\right)^{k+2} \frac{1}{x^{2(k+2)(1-\epsilon)}} \\
I_2 &= \left(\frac{\Gamma(1-\epsilon)}{4\pi^{2-\epsilon}}\right)^{k+3} \frac{1}{x^{2k(1-\epsilon)}} \int d^{4-2\epsilon}y \; \frac{1}{y^{2(1-\epsilon)}(y-x)^{4(1-\epsilon)}} \\
I_3^{\alpha\dot{\alpha}} &= \left(\frac{\Gamma(1-\epsilon)}{4\pi^{2-\epsilon}}\right)^{k+3} \frac{1}{x^{2(k-1)(1-\epsilon)}} \left(-i\partial^{\alpha\dot{\alpha}} \frac{1}{x^{2(1-\epsilon)}}\right) \int d^{4-2\epsilon}y \; \frac{1}{y^{2(1-\epsilon)}(y-x)^{4(1-\epsilon)}} \\
I_4 &= \left(\frac{\Gamma(1-\epsilon)}{4\pi^{2-\epsilon}}\right)^{k+4} \frac{1}{x^{2(k-1)(1-\epsilon)}} \int d^{4-2\epsilon}y \, d^{4-2\epsilon}z \left(-i\partial^{\alpha\dot{\alpha}} \frac{1}{z^{2(1-\epsilon)}}\right) \times \\
&\qquad\qquad \times \left(i\partial_{\alpha\dot{\alpha}} \frac{1}{(z-x)^{2(1-\epsilon)}}\right) \frac{1}{[y\,(y-z)\,(y-x)]^{2(1-\epsilon)}} \\
I_5^{\alpha\dot{\alpha},\beta\dot{\beta}} &= \left(\frac{\Gamma(1-\epsilon)}{4\pi^{2-\epsilon}}\right)^{k+4} \frac{1}{x^{2(k-1)(1-\epsilon)}} \int d^{4-2\epsilon}y \, d^{4-2\epsilon}z \left(-i\partial^{\beta\dot{\alpha}} \frac{1}{z^{2(1-\epsilon)}}\right) \times \\
&\qquad\qquad \times \left(i\partial^{\alpha\dot{\beta}} \frac{1}{(z-x)^{2(1-\epsilon)}}\right) \frac{1}{[y\,(y-z)\,(y-x)]^{2(1-\epsilon)}} \\
I_6^{\alpha\dot{\alpha}} &= \left(\frac{\Gamma(1-\epsilon)}{4\pi^{2-\epsilon}}\right)^{k+4} \frac{1}{x^{2(k-2)(1-\epsilon)}} \left(-i\partial_{\beta\dot{\beta}} \frac{1}{x^{2(1-\epsilon)}}\right) \int d^{4-2\epsilon}y \, d^{4-2\epsilon}z \left(-i\partial^{\beta\dot{\alpha}} \frac{1}{z^{2(1-\epsilon)}}\right) \times \\
&\qquad\qquad \times \left(i\partial^{\alpha\dot{\beta}} \frac{1}{(z-x)^{2(1-\epsilon)}}\right) \frac{1}{[y\,(y-z)\,(y-x)]^{2(1-\epsilon)}} \qquad (14)
\end{aligned}
$$

With our conventions we have

$$
\begin{aligned}
x^2 &\equiv 2x^{\alpha\dot\alpha}x_{\alpha\dot\alpha} \\
x^{\alpha\dot\alpha}x^{\beta}_{\dot\alpha} &= \frac{1}{4}C^{\beta\alpha}x^2 \\
i\partial^{\alpha\dot\alpha}\frac{1}{x^{2n}} &= -4n\frac{ix^{\alpha\dot\alpha}}{x^{2(n+1)}}\,.
\end{aligned}
\tag{15}
$$

The I_4, $I_5^{\alpha\dot\alpha,\beta\dot\beta}$ and $I_6^{\alpha\dot\alpha}$ contributions are finite. They can be computed (see for example [15]) evaluating the following integral in the $\epsilon \to 0$ limit

$$
\begin{aligned}
&\left(\frac{\Gamma(1-\epsilon)}{4\pi^{2-\epsilon}}\right)^5 \int d^{4-2\epsilon}y\, d^{4-2\epsilon}z \left(i\partial^{\alpha\dot\alpha}\frac{1}{z^{2(1-\epsilon)}}\right)\left(i\partial^{\beta\dot\beta}\frac{1}{(z-x)^{2(1-\epsilon)}}\right)\frac{1}{[y(y-z)(y-x)]^{2(1-\epsilon)}} \\
&\qquad\qquad \longrightarrow -\frac{1}{12(2\pi)^6}\frac{1}{x^4}\left(C^{\alpha\beta}C^{\dot\alpha\dot\beta}+4\frac{x^{\alpha\dot\alpha}x^{\beta\dot\beta}}{x^2}\right).
\end{aligned}
\tag{16}
$$

We notice that these are the terms that one would need to worry about maintaining supersymmetry via regularization: indeed supersymmetric regularization would tell us to use the dimensional reduction rule $C^{\alpha\beta}C_{\alpha\beta} = 2$ as opposed to the dimensional regularization rule $C^{\alpha\beta}C_{\alpha\beta} = 2-\epsilon$. Of course, being these integrals finite, any rule is a good rule. The I_2 and $I_3^{\alpha\dot\alpha}$ contributions are divergent but do not contain potentially dangerous contractions of indices. Thus one computes the integrals and subtracts subdivergences in a standard manner. With an overall common factor

$$
\frac{1}{(2\pi)^{2(k+2)}}
\tag{17}
$$

we obtain the following finite results, up to integrations by parts

$$
\begin{aligned}
I_1 &= \frac{1}{x^{2(k+2)}} = \frac{1}{4k\,(k+1)}\Box\frac{1}{x^{2(k+1)}} \\
I_2 &= -\frac{G\,(x)}{4}\frac{1}{x^{2(k+1)}} \\
I_3^{\alpha\dot\alpha}\bar D_{\dot\alpha}D_\alpha &= -\frac{ix^{\alpha\dot\alpha}}{x^{2(k+2)}}G\,(x)\,\bar D_{\dot\alpha}D_\alpha = \frac{1}{4\,(k+1)}\left(i\partial^{\alpha\dot\alpha}\frac{1}{x^{2(k+1)}}\right)G\,(x)\,\bar D_{\dot\alpha}D_\alpha \\
&= -\frac{1}{4\,(k+1)}\frac{1}{x^{2(k+1)}}G\,(x)\,i\partial^{\alpha\dot\alpha}\bar D_{\dot\alpha}D_\alpha + \frac{1}{4(k+1)^2}\frac{1}{x^{2(k+1)}}i\partial^{\alpha\dot\alpha}\bar D_{\dot\alpha}D_\alpha \\
I_4 &= \frac{1}{2}\frac{1}{x^{2(k+1)}} \\
I_5^{\alpha\dot\alpha,\beta\dot\beta}i\partial_{\beta\dot\beta}\bar D_{\dot\alpha}D_\alpha &= -\frac{1}{12}\frac{1}{x^{2(k+1)}}i\partial^{\alpha\dot\alpha}\bar D_{\dot\alpha}D_\alpha - \frac{1}{3}i\partial^{\beta\dot\beta}\left(\frac{x_{\beta\dot\alpha}x_{\alpha\dot\beta}}{x^{2(k+2)}}\right)\bar D^{\dot\alpha}D^\alpha \\
&= -\frac{1}{4\,(k+1)}\frac{1}{x^{2(k+1)}}i\partial^{\alpha\dot\alpha}\bar D_{\dot\alpha}D_\alpha \\
I_6^{\alpha\dot\alpha}\bar D_{\dot\alpha}D_\alpha &= -\frac{1}{12}\frac{1}{x^{2k}}\left(i\partial_{\beta\dot\beta}\frac{1}{x^2}\right)\left(C^{\alpha\beta}C^{\dot\beta\dot\alpha}+4\frac{x^{\beta\dot\alpha}x^{\alpha\dot\beta}}{x^2}\right)\bar D_{\dot\alpha}D_\alpha
\end{aligned}
\tag{18}
$$

$$= -\frac{1}{12\,(k+1)}\frac{1}{x^{2(k+1)}}i\partial^{\alpha\dot{\alpha}}\bar{D}_{\dot{\alpha}}D_{\alpha} + \frac{1}{12\,(k+1)}\frac{1}{x^{2(k+1)}}i\partial^{\alpha\dot{\alpha}}\bar{D}_{\dot{\alpha}}D_{\alpha} = 0$$

where we have defined $G\,(x) = \alpha - \log x^2$ with α a scheme dependent constant.

Having performed the $D-$algebra and computed the resulting bosonic integrals, in order to obtain the complete g^2 contribution to the two-point correlator we need insert the color and symmetry factors for each diagram in figure 1.

We notice that a non trivial check of the $D-$algebra can be done specializing the result for chiral primary CPO-type operators in which case we know that the total answer must give a vanishing result. We consider such an operator, e.g.

$$\mathcal{C} \equiv \bar{\Phi}_1 \Phi^{(2}\Phi^3 \cdots \Phi^{3)} \tag{19}$$

where the total number of fields is again $k+1$. Then if we compute the correlator

$$\left\langle \mathcal{C}\,(0)\,\bar{\mathcal{C}}\,(z)\right\rangle \tag{20}$$

to order g^2, we will have the same type of diagrams as in figure 1 with the result of the $D-$algebra given in (13). The color and symmetry factors $f_{(i)}$ for the CPO operators in (19) are easily computed leading to

$$f_{(a)} = f_{(b)} = -f_{(c)} = f_{(d)} = f_{(d')} = f_{(e)} = f_{(e')} \tag{21}$$

It is immediate to check that using (21) in (13) the contributions from the various diagrams exactly cancel.

Now we turn to the computation of the color and symmetry factors $F_{(i)}$ for the Konishi-type correlator. We find

$$F_{(a)} = F_{(b)} = \frac{k}{2}F_{(c)} = F_{(d)} = F_{(d')} = F_{(e)} = F_{(e')} = 2g^2 N F_{\rm tree} \tag{22}$$

where we have assumed gauge group SU(N), and

$$F_{\rm tree} = (k+2)\,(k-1)!\,\delta^{j_1}_{(j'_1} \cdots \delta^{j_{k-1}}_{j'_{k-1})}\,\mathrm{Tr}\left(T_{(a_1}\cdots T_{a_{k+1})}\right)\mathrm{Tr}\left(T_{(a_1}\cdots T_{a_{k+1})}\right) \tag{23}$$

is the color and symmetry factor which appears in the tree-level computation in (12). Now we can assemble the various contributions for the Konishi-type two-point function: with an overall factor

$$\frac{1}{(2\pi)^{2(k+2)}}2g^2 N F_{\rm tree} \tag{24}$$

we have

$$\begin{aligned}\left(1+\frac{2}{k}\right) J\left(0,\theta,\bar{\theta}\right)\Big[&I_1\,(x) - 2I_2\,(x)\left(\bar{D}^2D^2 + i\partial_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}D^{\alpha}\right) + 2\,k\,I_3^{\alpha\dot{\alpha}}\,(x)\,\bar{D}_{\dot{\alpha}}D_{\alpha} + \\ &+ I_4\,(x)\,\bar{D}^2D^2 - I_5^{\alpha\dot{\alpha},\beta\dot{\beta}}\,(x)\,i\partial_{\beta\dot{\beta}}\bar{D}_{\dot{\alpha}}D_{\alpha} + \\ &+ (k-1)\,I_6^{\alpha\dot{\alpha}}\,(x)\,\bar{D}_{\dot{\alpha}}D_{\alpha}\Big]\bar{J}\left(x,\theta,\bar{\theta}\right)\end{aligned} \tag{25}$$

Notice that the sum of the diagrams $(a), (b), (d), (d'), (e), (e')$ which contain a vector propagator reproduces the contributions from the graph (c) which contains a chiral vertex. Now using the results obtained in (14) we get the $\mathcal{O}\left(g^2\right)$ Konishi-like correlator

$$\begin{aligned} \mathcal{K}_{g^2}(0,z) = & \frac{g^2 N}{(2\pi)^{2(k+2)}} F_{\text{tree}} \frac{k+2}{k} \left[\left(\bar{D}^2 D^2 + \frac{1}{k+1} i\partial_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} D^{\alpha} \right) \frac{1}{x^{2(k+1)}} G(x) + \right. \\ & + \bar{D}^2 D^2 \frac{1}{x^{2(k+1)}} + \frac{3k+1}{2(k+1)^2} i\partial_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} D^{\alpha} \frac{1}{x^{2(k+1)}} + \\ & \left. + \frac{1}{2k(k+1)} \Box \frac{1}{x^{2(k+1)}} \right] \delta^4(\theta) \,. \end{aligned} \tag{26}$$

Now we can go back to the expression in (6) where we have the expected result for the correlator up to $\mathcal{O}\left(g^2\right)$. A direct comparison of (12) and (26) with (6) gives the $\mathcal{O}\left(g^2\right)$ value of the anomalous dimension

$$\gamma = \frac{k+2}{k} \frac{g^2 N}{4\pi^2} \tag{27}$$

and the normalization of the two-point function

$$f_{\mathcal{O}} = \frac{1}{(2\pi)^{2(k+1)}} F_{\text{tree}} \left[1 + \frac{k+2}{k} \frac{g^2 N}{4\pi^2} (\alpha + 1) \right] \tag{28}$$

where α is the scheme dependent constant appearing in the function $G(x)$. Let's notice that for $k = 1$ the value of γ in (27) correctly reproduces the known anomalous dimension of the Konishi operator [16]. Indeed, for $k = 1, 2$ our operators (2) do not suffer by the mixing problem, so in these two cases the values in (27) are eigenvalues of the dilation operator (cfr. [13]).

We notice that the $\mathcal{O}\left(g^2\right)$ correction to the normalization of the two-point function is proportional to the value of the anomalous dimension (this result, while being obvious for the scheme dependent part in (28), was instead not expected for the $\alpha-$independent term in the one-loop contribution). This suggests that these two objects are related to each other and, in particular, that protection of the dimension implies also non-renormalization of the two-point function. This is in fact the case for the CPO-like operators. The way the final answer rearranges itself suggests that the result might be extended to higher orders.

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